Giovanni Anastasio

Propositional Calculus Problem Set

1. **Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus**
    a. P
    b. P′
    c. P″
    d. Q
    e. Q′
    f. Q″
    g. R
    h. R′
    i. R″
2. **Propositional calculus in terms of a formal system**
    a. **How many axioms in the formal system?**
        i. Infinite
    b. **How many rules in the formal system?**
        i. 9
    c. **What are the names that he gives to these rules?**
        i. Joining Rule
        ii. Separation Rule
        iii. Double-Tilde Rule
        iv. Fantasy Rule
        v. Carry-Over Rule
        vi. Rule of Detachment
        vii. Contrapositive Rule
        viii. De Morgan's Rule
        ix. Switcheroo Rule
    d. **What is the one rule that you absolutely must use if you are to derive a theorem in this system?**
        i. Fantasy
3. **Write down each of the rules of the system, just as Hofstadter does on page 187.**
        i. Joining Rule: If x and y are theorems, then < x ∧ y > is a theorem
        ii. Separation Rule: If < x ∧ y > is a theorem, then both x and y are theorems.
        iii. Double-Tilde Rule: The string '~~' can be deleted from any theorem. It can also be inserted into any theorem, provided that the resulting string is itself well-formed
        iv. Fantasy Rule: If y can be derived when x is assumed to be a theorem, then < x ⊃ y > is a theorem.
        v. Carry-Over Rule: Inside a fantasy, any theorem from the "reality" one level higher can be brought in and used

vi. Rule of Detachment: If x and < x ⊃ y > are both theorems, then y is a theorem.
vii. Contrapositive Rule: < x ⊃ y > and < ~ y ⊃ ~x > are interchangeable.
viii. De Morgan's Rule: < ~x ∧ ~y > and ~< x ∨ y > are interchangeable.
ix. Switcheroo Rule: < x ∨ y > and < ~ x ⊃ y > are interchangeable.

4. **Derive: < < < P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R > > >**

[	Push
< < P ∧ Q > ∧ R >	Premise
< P ∧ Q >	Separation
P	Separation
R	Separation
Q	Separation
< Q ∧ R >	Joining
< P ∧ < Q ∧ R > >	Joining
]	Pop
< < < P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R > > >	Fantasy

5. **Derive: < < < P ∨ Q > ⊃ < Q ∨ P > >**

[	Push
< Q ∨ P >	Premise
P	Separation
Q	Separation
< P ∨ Q >	Joining
]	Pop
< < < P ∨ Q > ⊃ < Q ∨ P > >	Fantasy

6. **Derive a theorem in the propositional calculus that you think is a little bit interesting. Derive:** < < Q ∧ R >⊃< ~~P> >

[	Push
< Q ∧ R >	Premise
P	Separation
R	Separation
Q	Separation
< ~~P>	Double-Tilde
]	Pop
< < Q ∧ R >⊃< ~~P> >	Fantasy

7. **As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the**

method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.

a. A truth table is mathematical table used in logic that shows the logical combination of input values of different variables and the logical operators.
   i. < Q ∧ P >

Q	P	Q ∧ P
T	T	T
F	T	F
T	F	F
F	F	F

b. Since a truth table shows if something is a logical statement, by checking a WFF theorem using a truth table you can see if it is a theorem in propositional calculus based on if the argument provided is valid.

8. **Using the truth table-based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm referring to the section on Gantos Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem: < < < P ⊃ Q > ∧ < ~ P ⊃ Q > > ⊃ Q >**

P	**Q**		<	<	<	**P**	⊃	**Q**	>	∧	<	~	**P**	⊃	**Q**	>	>	⊃	**Q**	>
T	**T**					**T**	**T**	**T**		**T**		**F**	**T**	**T**	**T**			**T**	**T**	
T	**F**					**T**	**F**	**F**		**F**		**F**	**T**	**T**	**F**			**T**	**F**	
F	**T**					**F**	**T**	**T**		**T**		**T**	**F**	**T**	**T**			**T**	**T**	
F	**F**					**F**	**T**	**F**		**F**		**T**	**F**	**F**	**F**			**T**	**F**	

9. **Choose another interpretation for P and Q in Ganto's statement one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a sentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.**
   a. P: It is cloudy.
   b. Q: There will be rain
   c. Sentence: If it is cloudy, there will be rain; If it not cloudy, there will still be rain.
10. **Write down in a meaningful manner, in no more than a few sentences, what do you think is the salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and derived Rules.**
    a. What I believe the salient idea of this section is as people do derivations in propositional calculus they tend to take shortcuts and use previous derivations as a mold for the current one they are working on. This is considered an accepted

procedure as you are working outside the system and you are always led to a new theorem.

11. **Write down in a meaningful manner, in no more than a few sentences, what do you think is the salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.**
    a. What I believe the salient idea of this section is instead of constantly stepping out of the system which is a drawback of using shortcuts. You instead create a metatheory, which is a derived shortcut that you have made a theorem. This way you can speed up the process of deriving theorems of propositional calculus.
12. **Write down in a meaningful manner, in no more than a few sentences, what do you think is the salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.**
    a. What I believe the salient idea of this section is that concepts in propositional calculus may be simple, but they are very precise, making propositional calculus so appealing to mathematicians. The other main aspect of this section talks about two other reasons why propositional calculus is so appealing to mathematicians. Those reasons being that, propositional calculus can be studied for its own properties. The other reason being that it can be easily extended to include other fundamental aspects of reasoning.
13. **Write down in a meaningful manner, in no more than a few sentences, what do you think is the salient idea that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.**
    a. What I believe the salient idea of this section is that there are important similarities and differences between what a proof is and what a derivation is. A proof is informal or a product of human thought, written in a human language. Although a proof requires complex features of thought that are difficult to model logically. Where a derivation looks to accomplish the same idea as a proof but by using a logical structure whose methods are all explicit and simple.
14. **Write down in a meaningful manner, in no more than a few sentences, what do you think is the salient idea that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.**
    a. What I believe the salient idea of this section is dealing with contradictions as a human verse a machine/program. When a human faces a contradiction, they begin to question their morals and way of thinking and try to reason with themselves. Where when a machine faces a contradiction, they fail because they are incapable of adjusting to unknown input, if they were not properly programmed to do so.
15. **In one paragraph, write your reaction to this chapter.**
    a. I found this chapter rather interesting and informing as I had little prior knowledge on propositional calculus. It was nice to work with a formal system with such unique rules. I especially like working with the fantasy rule as it really shows the importance of working inside and outside of a system. I found working with the derivations to be tough at first, but with further practice and with the help of the shortcuts/metatheories I believe it will start to become a lot easier. Lastly, I

found the final section of this chapter to be my favorite because it deals with how humans can reason with a contradiction, where most computers would just crash and have no way of dealing with it.